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Galactic magnetic field irregularities and their effect on cosmic ray propagation at energies above 10^{17} eV

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Abstract. Astronomical data have been examined with regard to the characteristics of the irregular component of the magnetic field in the galaxy. A model has been formulated for the irregular field, and cosmic ray trajectories have been traced through the aggregate field. Comparison of experimental data on the arrival directions of extensive air showers with the predicted distributions shows that if the primary cosmic rays are protons then the majority between about 7×10^{17} eV and 10^{19} eV must be of extragalactic origin. Outside these limits the present arrival direction data do not allow distinction to be drawn between galactic and extragalactic origin.

1. Introduction

In a recent paper Karakula *et al* (1972, to be referred to as I) gave the results of computations on the trajectories of cosmic ray protons in the galaxy. A number of models for the large scale galactic field were considered and a study was made of the expected anisotropy of arrival directions at the earth if cosmic rays are of galactic origin. The field models used were all rather simplified to the extent of dealing only with large scale coherent distributions; in the present work we present a preliminary analysis of the effect of including field irregularities.

The specific object of the work described in the paper is the same as that in I, that is to examine the magnitude of the component of the high energy cosmic radiation that can have come from galactic sources by comparing the observed anisotropy of arrival directions with that expected for various fractions of the flux of galactic origin. In particular, we are interested in seeing whether there is an allowable choice of astrophysical parameters (distribution of coherent galactic field, field irregularity pattern etc) which will enable the conclusion to be drawn that there is the possibility of all cosmic rays coming from galactic sources.

In I a model was considered in which cosmic rays come from sources distributed uniformly throughout the galaxy and with coherent field distributions alone and it was shown that the predicted anisotropy is large at momenta below about 10^{18} eV, due to magnetic field trapping, and above 10^{19} eV, where magnetic deflexions are small, due to the lack of spherical symmetry of the source distribution with respect to the earth (ie the fact that the galaxy is a flattened spiral rather than a sphere).

The effect of field irregularities is to reduce the predicted anisotropy at the lower momenta. At very low momenta, $\lesssim 10^{14}$ eV/c, it is presumed that very small scale irregularities are responsible for the observed isotropy. By this is meant a scale of about 1 pc and an irregular field component of a few microgauss so that cosmic rays essentially diffuse through the galaxy at these momenta. Indeed, there is the suggestion that the steepening of the primary spectrum above about 10^{15} eV/c may be due to the onset of momentum-dependent diffusion which arises when the collisions between the particles and the field irregularity 'cells' cease to lead to randomly directed scattering. This region will be the subject of a later paper—in the present work we restrict attention to the scale of irregularities which effect the particles of highest momentum. The concept of field 'cells' is used, that is, regions of space in which the irregular component of the field can be considered to be uniform. Successive cells then have their own fields directed in random directions. Presumably there is a frequency distribution of cell sizes, cell separations and irregular field magnitudes and our object is first to examine astronomical data to see what information can be gleaned about these quantities.

2. Existence and magnitude of field irregularities

2.1. General remarks

The very nature of the galaxy, with its lack of smooth overall geometrical structure, the presence of irregular clouds of gas and dust, and the uneven distribution of stars, leads to the expectation of magnetic field irregularities. This is borne out immediately by a comparison of the average line of sight magnetic field from studies of the Faraday rotation measures of radiation from pulsars ($\sim 3\ \mu$G) with that from synchrotron radiation analyses, which indicate higher values ($\simeq 6\ \mu$G). Of course, there are uncertainties in the parameters which contribute to these values, but there does appear to be a marked disparity in the mean fields.

The problem in the analysis of the field irregularities is that different methods give rise to different samples of the field. Thus, a study of stellar polarization samples the field in those regions occupied by interstellar dust grains, rotation measures sample the electron distributions and synchrotron radiation relates to those regions containing fast electrons.

In what follows, an examination of evidence on discrete clouds of gas and dust is followed by a discussion of each of the methods in turn.

2.2. Galactic clouds

The presence of clouds of ionized and unionized gas is a prominent feature of the galaxy. Spitzer (1968) has given a useful summary of the information on the gas clouds; the parameters of a 'standard' cloud are: radius 7 pc, number in line of sight per kpc = 8, density of hydrogen = 10 atoms/cm^3, density of heavy ions = 5×10^{-3} ions/cm^3. In addition there is evidence for a class of 'large' clouds having a mean radius of the order of 35 pc.

There is evidence too for the presence of clouds in the dust population of the galaxy. As would be expected, there is often a connection between the dust and gas clouds, indeed dust particles are formed, presumably, from the condensation of gas molecules and the rate of condensation increases rapidly with gas density. We think it is reasonable to assume, to the accuracy required in the present work, that the 'standard' dust cloud has similar dimensions to that of a 'standard' gas cloud, that is, radius of approximately 7 pc and mean separation about 125 pc (by 'separation' is meant the mean distance between encounters in a straight line). Some of the clouds of dust and gas will occur in the same place, others will not.

Turning to the distribution of electrons in the galaxy there is the well known concentration in the HII regions. From studies of the emission and absorption of radio waves, Spitzer (1968) concludes that the fraction of space occupied by these regions is about 0·02 and this means that the sizes and separations of the clouds may not differ too much from those of the HI clouds. In the treatment of Faraday rotation given later it is assumed that the bulk of rotation arises in the concentrated regions—in fact there will be significant rotation in the space between the clouds but its neglect is not expected to alter the conclusions greatly.

To conclude this examination of clouds of dust, gas and ionized matter, it is sufficiently accurate for the present purpose to assume the presence of clouds of each, of somewhat similar dimensions (linear dimension of approximately 15 pc and separation about 100 pc). So far as the field irregularities are concerned it is reasonable to assume that each is the site of a cell of field; as will be discussed later, the problem is to decide whether cells of field are also present between the clouds in regions where they cannot be detected by the usual means.

The procedure now is to examine the results of the various methods of studying the field from the standpoint of seeing the extent to which the cloud parameters just given can be confirmed and of determining the magnitude of the irregular field.

2.3. *Stellar polarization*

Much of the early evidence for a coherent magnetic field in the galaxy, leading to the formulation of specific field models, was associated with studies of stellar polarization. In this work it was assumed that the measured polarization arises from the scattering of stellar radiation by interstellar dust grains in the manner suggested by Davis and Greenstein (1951). It is necessary to stress this assumption in view of controversy over the manner in which the polarization occurs (see, eg, the work of Wickramasinghe 1969 and Martin 1972). However, there seems to be sufficient probability of the Davis–Greenstein process being applicable to accept field estimates from such an analysis.

The measured polarizations can also give information about the irregular field and some work has been done in this area, notably by Jokipii *et al* (1969). These workers used the experimental data of Behr (1959) and a simple analysis which examined the variance of the polarization along and perpendicular to the direction of the coherent field, to show that the correlation length for the irregular field is of the order of 150 pc. By 'correlation length' is meant the average distance over which the detected irregular field changes in direction by a large amount ($\sim \pi/2$).

In the present work we have made an independent analysis of the problem using the more extensive data now available and a different treatment to that of Jokipii *et al* (1969). The basic data are the plots of the electric vectors of light from nearly 7000 stars given by Mathewson and Ford (1970). These plots, which refer to stars within distance intervals 0–50, 50–100, . . . , 2000–4000 pc, give the magnitude and direction of the electric vector so that if the polarizing dust grain density is known throughout the space in question a detailed map of the field perpendicular to the line of sight can be made (to be precise, the square of the field, for the most likely case of the grains not being completely aligned). In fact the density variation is not known in general but to the accuracy needed in the present work it is sufficient to assume a rather simple form, as will be described.

The present data indicate that the coherent field runs along the local spiral arm but do not definitely distinguish between a simple longitudinal field and a longitudinal field that reverses direction in crossing the galactic plane. We define a rectangular coordinate system with the x and y axes in the galactic plane. The x axis is the direction, galactic longitude $l = 180°$, and the y axis points towards $l = 270°$ (ie in the outward going spiral arm direction). The z axis passes through the sun and points to the galactic north pole. In the present work we consider two models of the coherent field, A and D. Model A is the reversing field model of Thielheim and Langhoff (1968) discussed in I. The z dependence of the magnitude of the field is $50z \exp\{-(z/0{\cdot}175)^2\}$ and the position of the sun is $z_0 = -0{\cdot}085$. All distances are in kiloparsecs. Model D is the simple longitudinal field with z dependence $-3{\cdot}8 \exp\{(-z/0{\cdot}24)^2\}$ and $z_0 = 0$. This corresponds to model B of I with the local helical perturbation omitted.

Returning to field irregularities, the polarization plots for directions close to the axis of the arm (ie $l = 90°$ and $270°$) give the magnitude of the field in question. Similarly, the polarization plots for $l = 0°$ and $180°$ give, mainly, the strength of the coherent field. In the analysis, we assume that the clouds of dust have equal diameter L_c and that they have mean separation L_s. It is assumed that the field within a cloud is the result of the superposition of the background coherent field and a random field that is uniform within a given cloud but randomly oriented. The ratio of magnitude of random to coherent field is taken to be a constant factor a.

Using the nomenclature of 'cells', to preserve generality, the polarization from an individual cell is

$$p = f\rho L_c H_\perp^2$$

if the field is weak enough for the grains not to be completely aligned, where ρ is the mean grain density in the cell of length L_c, $H_\perp$ is the perpendicular field and f is the polarization per unit field^2 and unit distance for unit density. Looking along the axis of the spiral arm ($l = 90°, 270°$ ie the y direction) where there is no perpendicular component of coherent field, the polarization of light from a star at distance d will be proportional to the square root of the number of cells encountered

$$p_R \simeq p\left(\frac{d}{L_s}\right)^{1/2},$$

that is,

$$p_R = f\rho_0 H_R^2 \left(\frac{d}{L_s}\right)^{1/2} L_c \qquad (1)$$

where ρ_0 is the grain density in clouds along the axis, assumed constant, and H_R is the mean perpendicular component of the random field strength. If now observation is made in the perpendicular direction, along $l = 0°$ and $180°$, the polarization will have components due mainly to the coherent field of magnitude:

$$p_c = f\frac{L_c}{L_s}\int_0^d H_c^2(x, y = 0, z = z_0)\rho(x, y = 0, z = z_0)\,dx.$$

The distribution of clouds of grains in the galaxy is not known with any accuracy but it is reasonable to assume that the grain density is given by $\rho(x, y = 0, z = z_0) \propto H_c$, that is, we assume that the grain density falls off above and below the galactic plane in the same way as the coherent field.

Thus

$$p_c \simeq \frac{f\rho_0}{H_c(0)}\frac{L_c}{L_s}\int_0^d H_c^3(x, y = 0, z = z_0)\,dx. \qquad (2)$$

From equation (2) a fit can be made to the experimental data of p_c against d and $f\rho_0 L_c/L_s$ can be derived. This can then be substituted in equation (1) and a comparison made with the data of p_R against d and an estimate of H_R^2 made.

The data are shown in figure 1. Figure 1(*a*) shows polarizations given by Mathewson and Ford (1970) for longitude ranges $-20°$ to $+10°$ and 160° to 190°, that is, approximately perpendicular to the spiral arm and in a plane parallel to the galactic plane. Figure 1(*b*) gives similar results for longitude ranges 60° to 90° and 270° to 300°. In both cases the points represent the median polarization for groups of measurements in the same distance range and the errors shown are those in which half of the values lie.

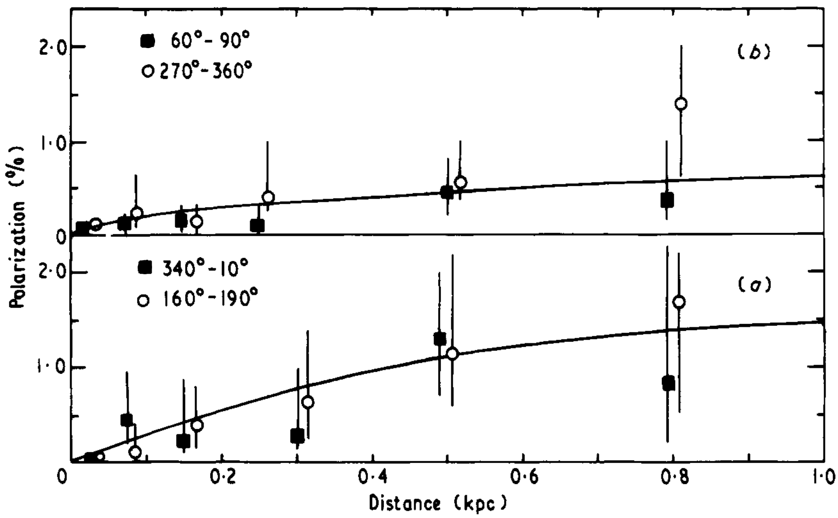

Figure 1. Polarization of starlight as a function of distance to the star in the directions (*a*) perpendicular to the spiral, (*b*) along the spiral axis. The lines show the best fits to expressions (2) and (1) respectively.

The line shown in figure 1(*a*) has the form given by equation (2) and from it $f\rho_0(L_c/L_s)$ follows as $6{\cdot}8\times10^{-5}\,\%\,\mathrm{pc}^{-1}\,\mu\mathrm{G}^{-1}$. The square root relation (1) fitting the data of figure 1(*b*) is shown and use of the value of $f\rho_0(L_c/L_s)$ gives

$$H_R^2\sqrt{L_s} = 300\,\mu\mathrm{G}^2\,\mathrm{pc}^{1/2}.$$

Allowing for the fact that polarization measures the perpendicular field we have $H_R = 0{\cdot}89a\,H_c$ and using $\overline{H}_c = 6\,\mu\mathrm{G}$ for the region near the axis of the spiral arm contributing to the measured values of p_R we have $a^2\sqrt{L_s} = 10$ for L_s measured in parsecs. In so far as the model value of the coherent field had been used in calculations earlier, insertion of $\overline{H}_c$ makes the expression for $a^2\sqrt{L_s}$ almost independent of its adopted magnitude.

The spread of polarization values about the mean shown in figure 1 can, in principle, be used to determine L_s and this is the method adopted by Jokipii *et al* (1969) and referred to earlier. Our data are not inconsistent with their value of 150 pc.

2.4. *Faraday rotation of extragalactic radio sources*

Jokipii and Lerche (1969) have analysed data on Faraday rotation in a rather similar fashion to their analysis of polarization data in order to examine the random component of the field. The analysis used the data of Berge and Seielstad (1967) which comprised about 70 sources having latitude $|b| > 10^0$ and the effect of a coherent field in addition to the random field was not considered.

In what follows the same analysis has been updated by including more recent source data from the summary of Mitton (1972), an attempt has been made to exclude values affected by large intrinsic rotations, and the coherent field has been allowed for.

The method is to calculate the quantity

$$\sigma_R^2 = \frac{1}{(n-1)} \sum (R_E - R_p)^2$$

for the sources in a selected band of galactic longitude, where R_E is the experimental rotation and R_p is the predicted value in the same direction for the adopted model for the coherent field. In so far as the analysis concerns the derivation of the variance of the rotation measures rather than the absolute field strength, a smooth distribution of electron density can be used in calculating R_p. Calculations were made for three different electron distributions, $n_e = 0{\cdot}05\ \mathrm{cm}^{-3}$, $n_e = 0{\cdot}09 \exp -(z/0{\cdot}121)^2$ (Davies 1969) and $n_e = 0{\cdot}05 \exp -(|z|/0{\cdot}152)\ \mathrm{cm}^{-3}$ (Prentice and ter Harr 1969). In fact the results are not very different and those to be given relate to $n_e = 0{\cdot}05\ \mathrm{cm}^{-3}$.

The results for bands of sin $|b|$ from 0·2 to 1·0 in steps of 0·1 are given in figure 2, where σ_R^2 has been multiplied by sin$|b|$. In an effort to reduce the effect of sources having high intrinsic rotation those with rotation measures greater than 2·5 standard deviations above the mean have been rejected from each band of sin$|b|$.

As for the polarization analysis, calculations were made with the two models of the coherent field, A and D, and again the differences were small.

The interpretation of plots of this type has been given by Jokipii and Lerche (1969). The authors use a simplified uniform disc model of the galaxy (thickness T) and define

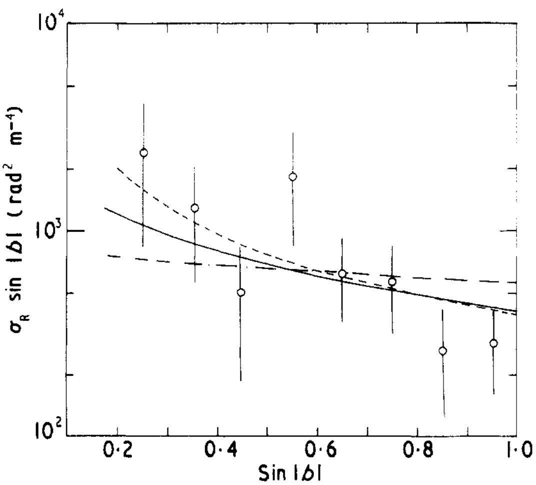

Figure 2. Dispersion of Faraday rotation measures of extragalactic radio sources. σ_R is the RMS dispersion for a given band of galactic latitude. Error bars show 90% confidence limits. Full curve $L/T = 1$, broken curve $L \gg T$, chain curve $L/T = 0{\cdot}2$.

a correlation length L which corresponds very roughly to our L_s if we now take this to be the mean separation between clouds of ionized gas. Then if $L \ll T$ one expects $\sigma_R^2 \propto \text{cosec}\,|b|$ while for $L \gg T$ one expects $\sigma_R^2 \propto (\text{cosec}\,|b|)^2$. The variations of $\sigma_R^2 \sin|b|$ with $\sin|b|$ for three values of L/T are shown in figure 2. Without knowledge of the mean square fluctuation of the product of electron density and field it is possible to compare only the shapes of these curves with the observed values. Hence the curves are individually normalized to the data. One may conclude then that $L/T \gtrsim 0{\cdot}8$, that is, $L_s \gtrsim 200$ pc. This is larger than the 125 pc obtained in §2.3 but the difference is not too surprising in view of the possibility of the presence, still, of significant intrinsic rotations.

2.5. *Faraday rotation of the radiation from pulsars*

Measurements of the rotation of the polarized radiation from pulsars are of great value in that there is less uncertainty in the adopted electron density in interstellar space. This arises because the line integral of the electron density alone can be found by determining the dispersion of the pulse.

An analysis has been carried out using the pulsar rotation measure data of Manchester (1972). After rejecting those pulsars that lie outside the local spiral arm or more than 250 pc from the galactic plane the 'measured' line of sight magnetic field to the remaining 14 pulsars has been compared with the values expected from the two coherent field models (A and D). Although, at first sight, the data, which show little evidence for field reversal at positive latitudes, would seem to favour the simple longitudinal field D, in fact the reversing field gives just as good a fit. This is because our specific model A has the sun 85 pc below the plane. The results for field D are shown in figure 3 presented in terms of H_D against H_{exp} where H_{exp} is the value derived from the pulsar measurements and H_D is that corresponding to model D. The best-fit line has a slope of 1·95, that is, the

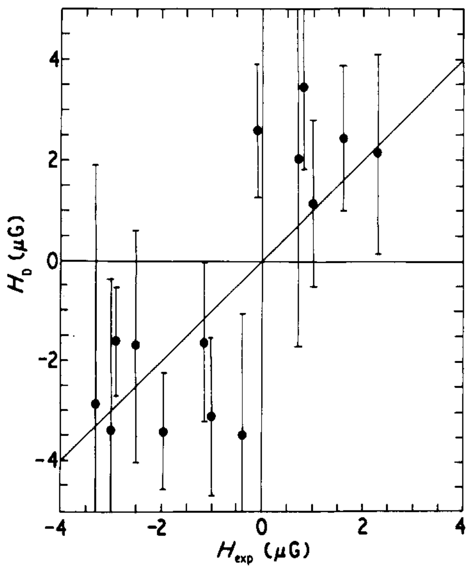

Figure 3. Comparison of predicted and observed values of the Faraday rotation measure from pulsars. H_D is the value expected for model D coherent field. H_{exp} is the experimental value.

field strength predicted by model D is about twice as large as that indicated by the pulsar rotation measurements. If this factor is correct it should be borne in mind that a scaling of the mean field is equivalent to scaling the effective cosmic ray momenta by the same factor.

If the spread of values in figure 3 about the best-fit line is attributed to the irregular fields a quantitive estimate of the scale of the irregularity can be estimated. The standard deviation of the spread in values of the line of sight magnetic field for a pulsar at a distance d is $0{\cdot}89\, a\, H_d(L_s/d)^{1/2}$ where H_d is the effective mean magnitude of the coherent field between the sun and the pulsar. The value of $a(L_s)^{1/2}$ which gives the optimum value of χ^2 for the experimental values has been determined. The result is $a(L_s/1\ \mathrm{pc})^{1/2} = 15$. The error flags in figure 3 give the one standard deviation limits corresponding to this value.

2.6. *Synchrotron radiation*

The mechanism here is the emission of radiation when cosmic ray electrons undergo magnetic deflexion in the galaxy.

A recent study is that by Anand *et al* (1968) and relates to surveys of radio brightness distributions: (i) the galactic ridge $l = 0°$, $b = +3{\cdot}6°$ and $-3{\cdot}6°$ (ie just above and just below the galactic nucleus), radio frequency 100–4000 MHz, (ii) the galactic anticentre, 10–500 MHz, and (iii) the north halo, excluding the north galactic spur, 10–400 MHz.

The method was to take the synchrotron spectrum for each direction and from it to calculate the shape of the interstellar electron spectrum as a function of the strength of the perpendicular component of the magnetic field $H_\perp$ (assumed constant). The electron spectrum was then normalized to the primary electron spectrum measured at the earth at 5 GeV, solar modulation being assumed negligible. Anand *et al* (1968) convert the mean perpendicular field $H_\perp$ to a mean field $\langle H\rangle$ by $\langle H\rangle = 1{\cdot}23 H_\perp$, that is, it is assumed that the field is randomly directed, with the result that $\langle H\rangle = 9{\cdot}5\ \mu$G in the direction of the galactic centre, 6 μG towards the anticentre and 2·5 μG towards the halo.

Some comment is needed about the value of 2·5 μG towards the halo. This value is small because it is assumed that a halo does in fact exist and the path length contributing towards the synchroton radiation is thus very much greater than if the halo is ignored. In our own treatment we assume that a significant halo does not exist and therefore for the present comparison a much higher field strength, or spectrum of relativistic electrons, from high latitude synchrotron studies would result. (The effects of a halo field on galactic cosmic ray anisotropies is discussed in I.) It is appropriate to compare the mean field derived from the synchrotron intensity in the direction of the anticentre (6 μG) with the appropriate average of $H_c(x, y = 0, z = z_0)$ given in figure 1. This latter is seen to be about 3·5 μG for models A and D, or if the pulsar rotation measures are to be believed, a value about one half of this. It is clear, therefore, that the synchrotron measures of the field are higher and this is strong evidence for the existence of significant irregularities. Taken at face value they indicate that the strengths of the regular and irregular fields are approximately the same. From studies of the polarization of the synchrotron radiation one can obtain information on only the local structure in the galactic field. Radiation from greater distances suffers from superposition effects and Faraday rotation between the observer and the source.

It should be remarked that the above synchrotron measures are much smaller than the value of 30 μG for the mean disc field given by Davies (1965). This arises because Anand *et al* take an emissivity from the disc (at 80 MHz) about a factor of four smaller and an electron intensity a factor of two higher than that used by Davies.

2.7. *Synthesis of the results on irregular fields*

In the simplest model of all, where it is assumed that dust and ionized gas are only present in clouds (both being present together) all the clouds are of the same diameter, and the irregular field is present only in the clouds, then the strength of the mean irregular field (aH_c) and the mean cloud separation (L_s) can be determined from a combination of the results we have derived from the various methods.

Stellar polarization:

present work	$a^2\sqrt{L_s} \simeq 10\,\mathrm{pc}^{1/2}$
Jokipii *et al* (1969) variance study	$L_s \simeq 150\,\mathrm{pc}$
Faraday rotation of extragalactic sources	$L_s \simeq 200\,\mathrm{pc}$
Faraday rotation of radiation from pulsars	$a\sqrt{L_s} \simeq 15\,\mathrm{pc}^{1/2}$.

Taking these relations together the result is that a is probably about unity and L_s is about 150 pc. This value of a is not inconsistent with the synchrotron results and with the measured Zeeman splitting of radiation from interstellar clouds. It is heartening to see that the value of L_s is in accordance with the intercloud spacing referred to in § 2.2.

3. Relevance to cosmic ray anisotropies

A reasonable model of the field irregularities can be seen to be one in which the irregularities comprise clouds of size about 10 pc, each containing a uniform field of strength roughly equal to that of the coherent field at that point but with random direction. The separation of the clouds might be about 100 pc (a little smaller than the 150 pc referred to in § 2.6 because some clouds of dust and ionized gas will not coincide).

A problem now concerns the magnitude of the irregular field in the space between the clouds where the methods do not sample it. It is unreasonable to assume that the field is quite uniform in this region and, for the purpose of calculating an upper limit to the effect on cosmic ray trajectories, calculations have been made for what might be regarded as the extreme case, namely that the irregular field takes on a new random direction every 10 pc. The anisotropies expected for the preferred model in which the field irregularities occur only every 100 pc are then derived by an approximate method.

3.1. *Details of the trajectory calculations*

The method is that adopted in I, that is, to determine the trajectories of antiprotons of given momentum starting from the earth in particular directions. The trajectories are followed, step by step, in the coherent plus irregular field until the particles leave the disc ($|z| > 300$ pc in this model). The total path length is then noted and it follows, as in I, that the expected intensity of cosmic rays of purely galactic origin in the particular direction is proportional to that length. There is a difference in the case of an irregular field, however, in that there is now no unique trajectory for a particular angle of incidence and the mean of a number of trajectories must be taken.

3.2. *Expected anisotropies*

In I detailed contour maps were given for the celestial sphere showing path lengths for various momenta. Experimental data on extensive air showers were then analysed and upper limits to the relative fluxes of energetic primary particles which could have come from galactic sources (for the model of uniform production throughout the galaxy) were derived. In the present work a simplified analysis has been made.

Trajectories have been determined for 25 directions, these directions being chosen and the resulting path lengths weighted to give equal representation to approximately equal areas of the galactic latitude–longitude sphere. Calculations were made for coherent field models A and D and for proton momenta from 6×10^{17} eV/c to 10^{19} eV/c.

The effect of the irregular field is a shortening in the overall mean path length and, particularly in the case of model D, a reduction in the variance of the individual lengths about the overall mean, especially at the lower proton energies.

An estimate of the effect of the irregular field on the expected anisotropy for particles of galactic origin has been made by examining the relationship between the actual path lengths with and without the irregular field (these conditions are abreviated $I_r = 1$ and $I_r = 0$ respectively). If, for a particular direction, x_i is the path length for $I_r = 0$ and y_i is the mean length for $I_r = 1$ then a scatter diagram x, y can be drawn. It is found that a straight line $y = bx + c$ is a reasonable fit for any one energy and it will be appreciated that the intercept c is a measure of the amount of randomization of path lengths that has occurred due to the irregular field. We can define the quantity

$$D = \frac{b\bar{x}}{b\bar{x}+c}$$

where $\bar{x}$ is the mean path length for $I_r = 0$ and D can be termed the 'randomization factor'. $D = 1$ means the irregular field has no effect while $D = 0$ implies that the irregular field has the effect of producing an isotropic distribution at the earth from what was, with $I_r = 0$ a most anisotropic one.

Figure 4 gives plots of D for the two field models. The calculations have not been extended below 6×10^{17} eV/c where the individual trajectories become too long to apply this method. Experimental upper limits to the percentage G of cosmic rays of

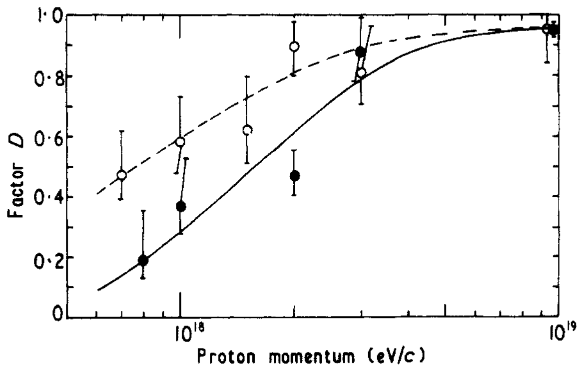

Figure 4. Randomization factor D, plotted against proton momentum for both models of the coherent field with $a = 1$. ○, broken curve field A; ●, full curve field D.

galactic origin, obtained as in I, for the case $I_r = 0$, can be converted to the corresponding upper limits for $I_r = 1$ by dividing by the factor D. Figures 5 and 6 show the experimental limits obtained from the Haverah Park (HP), Volcano Ranch (VR), and Sydney (S), EAS arrays for field models A and D respectively. The data from Volcano Ranch and Sydney are as used in paper I but the more recent Haverah Park results (Lapikens *et al*

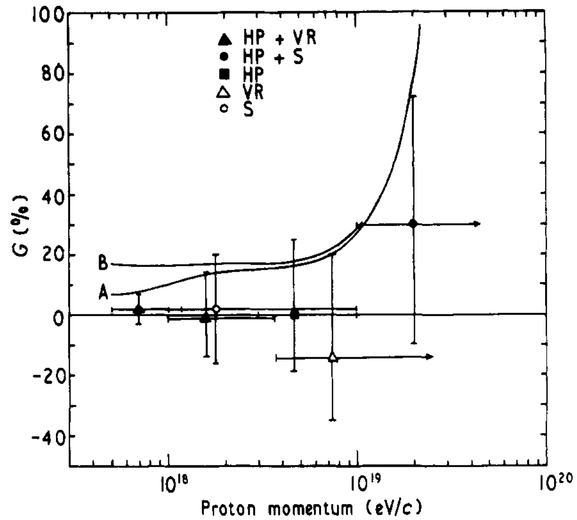

Figure 5. Upper limit (at 95% confidence level) to the percentage of protons G which come from galactic sources, for field A. A, regular field only; B, including irregularities.

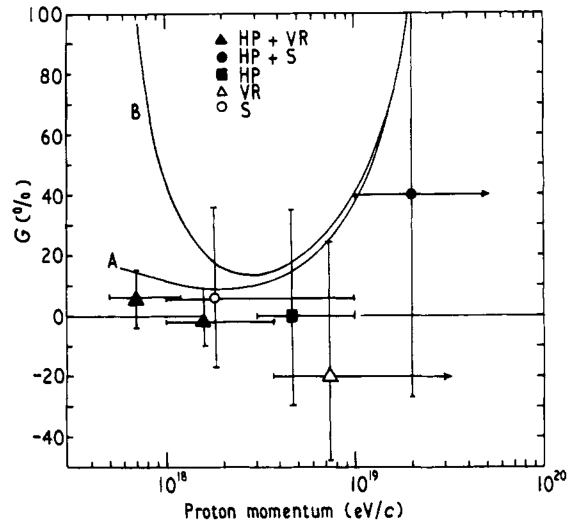

Figure 6. Upper limits as in figure 5 but for field D.

1971 and private communication) have now been included. The points plotted are the best-fit values of G and the error bars show the 95% confidence level limits. All the experimental data are consistent with isotropy which means that there is no lower limit to the galactic component. In each figure curve A is the upper limit to the fraction that could be galactic on grounds of isotropy alone for the case $I_r = 0$, and curve B is the limit for $I_r = 1$ obtained by applying the randomization factor D.

Figure 7 gives summary graphs of the 95% confidence level limits to G. The results for the model of the irregular magnetic field used in the calculations of D and having 10 irregular field cells per 100 pc are denoted by $L_s = 10$. The effect of this irregular

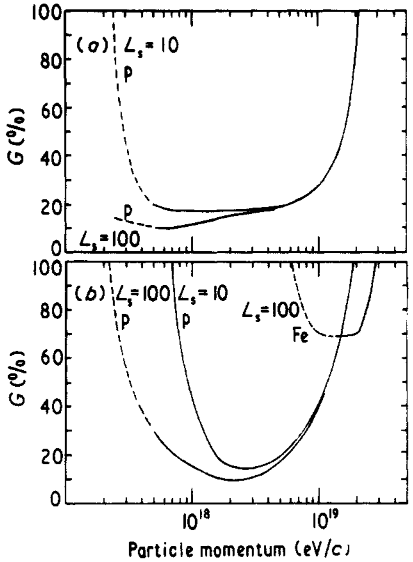

Figure 7. Summary of upper limits to G (at 95% confidence level). (a) Field A, (b) field D.

component on the model D field is that the predicted intensity distribution becomes sufficiently isotropic that G could be 100% for momenta below 7×10^{17} eV/c. On the other hand for field A the effect is not very significant down to the lowest calculated momentum. The dashed portion of the $L_s = 10$ curve is an estimate obtained by a simple linear extrapolation of the factor D to lower momenta.

Also shown are the upper limit curves for what may be regarded as the more reasonable model of the irregular field (§ 3) estimated in a very approximate way. In this model there is only one cloud of irregular field per 100 pc. This means that for the same amount of scattering per 100 pc a proton must have $1/\sqrt{10}$ times lower momentum. We then assume that the momentum dependence of the D factor scales by this factor and apply it again to the nonrandom field upper limits to obtain the curves denoted $L_s = 100$.

3.3. *Discussion of results and conclusions*

Inspection of figure 7 shows that the effect of the irregular field, as calculated for the present extreme case of $L_s = 10$, is such as to enable most of cosmic rays below 7×10^{17} eV/c, for model D, to have been produced in the galaxy without producing a definitely detectable anisotropy. For model A the limiting momentum will be in the region of 2×10^{17} eV/c. With the more likely field configuration $L_s = 100$, however, the limiting momenta would be somewhat lower. For either model there is a region of at least a decade in momentum where 80 to 90% of cosmic rays must be extragalactic in origin. Beyond 10^{19} eV/c the total number of recorded showers rapidly becomes too small to set a meaningful limit.

All this assumes that the primaries are protons. If heavy primaries are invoked then the limiting momenta increase by a factor Z thus narrowing the momentum band in which at least some proportion of the cosmic rays must be extragalactic. For both fields and $L_s = 10$ there is no need to invoke extragalactic sources at all if the effective mean value of Z is greater than 10. For $L_s = 100$, field D requires at least some extra-galactic contribution even for the extreme case of all cosmic ray primaries being iron nuclei (figure 7). Field A, on the other hand, gives no upper limit at the 95 % confidence level to the galactic contribution for iron nuclei and $L_s = 100$. Although this field predicts a strong peak in the direction of the inward going spiral arm in the northern celestial hemisphere for iron nuclei of momentum 2×10^{19} eV/c there should be a rather isotropic distribution in the southern hemisphere. Most of the showers so far recorded with energy greater than 10^{19} eV have been seen by the Sydney array.

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